

Peasedown St John Primary School Progression in Calculations Nursery and Year R – 6



This calculation policy is a guide for all staff at Peasedown St John Primary School and forms part of the mathematics policy.

It is designed to be used alongside any teaching resources that teachers use.

At Peasedown St John Primary, we follow a Maths Mastery curriculum through the use of the National Curriculum Maths targets. This curriculum is supplemented by resources such as White Rose Maths, Third Space Learning, Testbase, CGP Maths textbooks and Target Your Maths textbooks.

Whilst the calculation policy guidance document is separated into year group phases, these are intended to be used only as a guide and it is the teachers' professional judgment as to when the pupils move on to the next phase.

As our understanding of mathematical mastery within the primary years develops at Peasedown St John School, this policy may be subject to change Peasedown St

John Primary School Progression in Calculations Year R – 6



	EYFS (40-60 months/ELG)	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Saying which number is one more than a given number. Finding the total number of items in two groups by counting all of them. Finding the total by starting at the bigger number and counting on.	Combining two parts to make a whole: part whole model. Starting at the bigger number and counting on. Regrouping to make 10.	Adding three single digits. Use of base 10 to combine two numbers	Column method regrouping Using place value counters (up to 3 digits).	Column method - regrouping. (Up to 4 digits)	Column method – regrouping (with more than 4 digits) Use of place value counters for adding decimals.	Column method regrouping. Abstract methods. Place value counters to be used for adding decimal numbers.

Subtraction	Taking away objects or drawings. Saying which number is one less than a given number. Subtracting two single digit numbers by counting back.	Taking away ones Counting back Find the difference Part whole model Make 10 using a tens frame	Counting back Finding the difference Part whole model Make 10 Use of base 10	Column method with regrouping (up to 3 digits using place value counters)	Column method – regrouping. (Up to 4 digits)	Column method – regrouping (with more than 4 digits) Abstract for whole numbers. Start with place value counters for decimals with the same amount of decimal places.	Column method - regrouping. Abstract methods. Place value counters for decimals- with different amounts of decimal places.
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Multiplication	Problem solving-doubling	Recognising and making equal groups. Doubling Counting in multiples -use cubes, Numicon and other objects in the classroom	Doubling Counting in multiples Repeated addition Arrays–showing commutative multiplication	Counting in multiples Repeated addition Arrays 2 digit $\times$ 1 digit using base 10 or place value counters	Column multiplication introduced with place value counters (2 and 3 digit multiplied by 1 digit)	Column multiplication (up to 4 digit numbers multiplied by 1 or 2 digits) Abstract only but might need a repeat of year 4 first	Column multiplication Abstract methods (multi digit up to 4 digits by a 2 digit number)
Division	Problem solving–halving and sharing	Sharing objects into groups Division as grouping e.g. I have 12 sweets and put them in groups of 3, how many groups?	Division Grouping Division Within arrays Repeated subtraction	Division with a remainder Short Division (2 digits by 1 digit using base 10 or place value counters)	Division with a remainder Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short Division (up to 4 digits by a 1 digit number interpret remainders)	Short division Long division with place value counters (up to 4 digits by a 2 digit number) Children should exchange into The tenths and hundredths column too

Nursery Addition Skills

Subitise to 3 • Count how many • Make numbers to 5 • Add 1 more (through songs and rhymes)

Subitise to 3
How many do you see?
Instantly see how many.



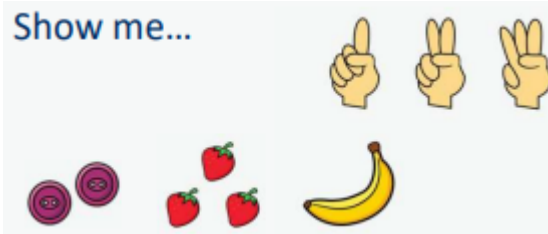
Count how many
How many are there? Begin to count objects using 1-1 correspondence.



Count out ... from a larger group.
E.g. Collect 3 beanbags for a game.



Make numbers to 5
Begin to link numerals to quantities. Start by showing 1, 2 and 3 using fingers.



Begin to link numerals to quantities.



Add 1 more
How many do I have now?
Through stories, songs and rhymes.

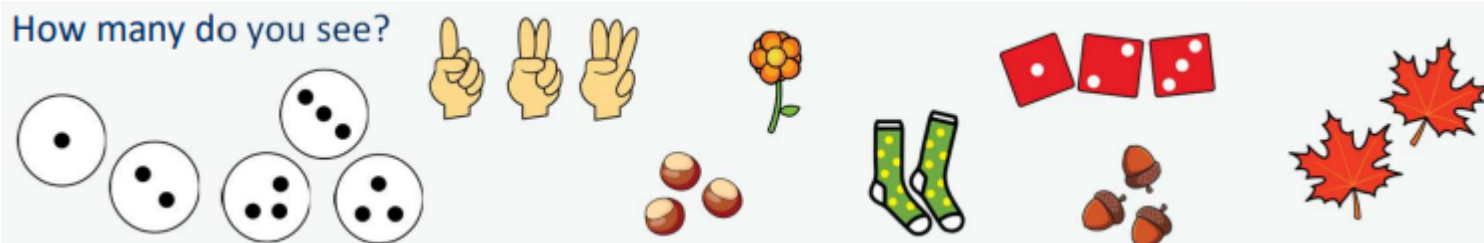


Nursery Subtraction Skills

Begin to have an understanding of numbers to 5 • We recommend focusing on noticing and representing small quantities, perceptual subitising and counting.

Subitise to 3

How many do you see?
Instantly see how many.



Count how many

How many are there? Begin to count objects using 1-1 correspondence.

How many are there?



Count out ... from a larger group.

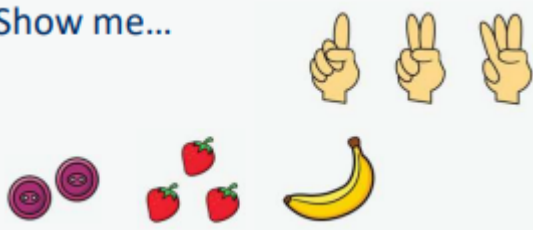
E.g. Collect a cup for everyone at the table.



Make numbers to 5

Begin to link numerals to quantities. Start by showing 1, 2 and 3 using fingers.

Show me...



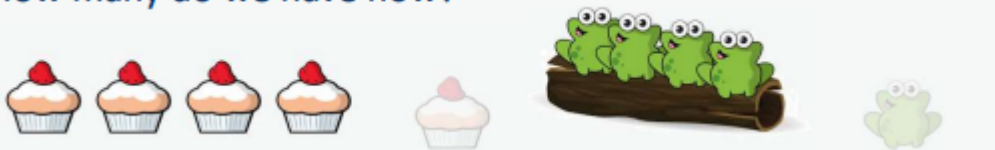
Begin to link numerals to quantities.



Take 1 away

How many do we have now?
Through stories, songs and rhymes.

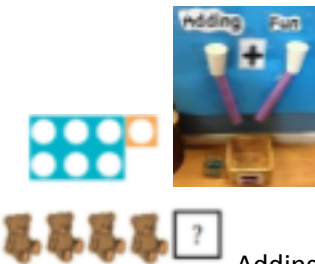
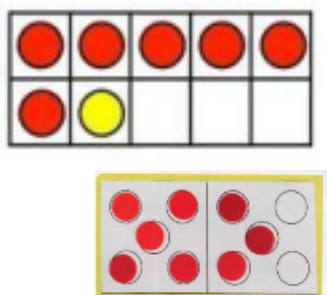
How many do we have now?

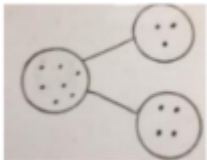
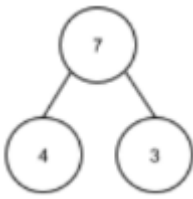


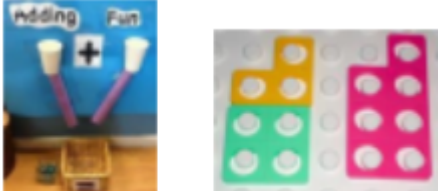
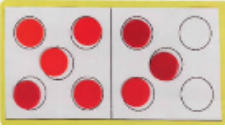
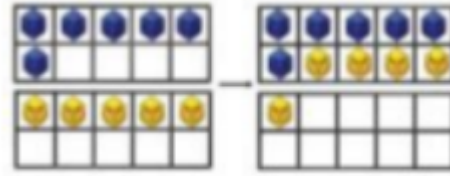
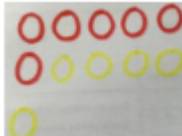
Peasedown St John Primary School Progression in Calculations Year R – 6

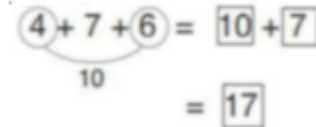
Addition

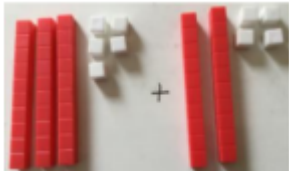
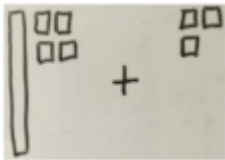
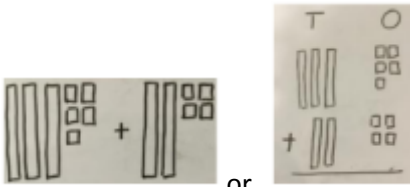
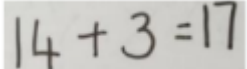
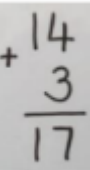
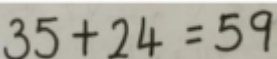
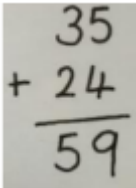
Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'

Objective & Strategy	Concrete	Pictorial	Abstract
Finding one more	Counting on using familiar resources and objects  Adding machine	Using ten frames and Hungarian ten frames with counters or drawing on ten frames. 	Introduction to + and = symbols to create equations (number sentences) $4 + 1 = 5$

Combining two parts to make a whole (aggregation)	Combining two parts to make a whole (use a range of resources from the classroom)  Rekenrek Adding machine	Objects initially used on a part-whole model and then represented using dots or circles. The part-whole model is presented in different Orientations. 	Four is a part Three is a part The whole is seven  $4 + 3 = 7$
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Counting on (augmentation)	Concrete resources may not change, but children begin to count on from the largest number  Adding machine 	Children draw circles for the amount to be added on and count on using the circles: $8 + \bigcirc \bigcirc \bigcirc \bigcirc =$	Recording as equations (number sentences) $8 + 4 = 12$
Re-grouping to make 10	Using ten frames and counters 	Children draw their own representation of ten frames and counters. Start with the larger number and use the smaller number to make 10. 	Children record as equations and begin to understand equality and commutativity $6 + 5 = 11$ $6 + 5 = 5 + 6$ $6 + 5 = 7 + 4$ important step

Adding 3 single digits	Children look for known number bonds, using their knowledge of commutativity. Ten frames may also be used for this step. 	This stage is not represented pictorially, as the focus is on recall of known number facts. Concrete objects are used initially to demonstrate commutativity.	Look for known number bonds and combine these numbers first before adding on the remaining number 
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Combining 2 numbers – no carrying or Exchanging Prior knowledge needs to be secure: making numbers using dienes	Children represent the numbers to be added using Base 10 and then add together first the ones then the tens. TO + O using Base 10 (Dienes)  TO + TO using Base 10 	After using Base 10 practically, children draw it out, remembering to solve the addition by adding ones and then tens. TO + O drawing Base 10 (Dienes)  TO + TO drawing Base 10  or	Record horizontally initially, moving onto column addition, adding ones and then tens.  When ready, moving onto:   When ready, moving onto: : 
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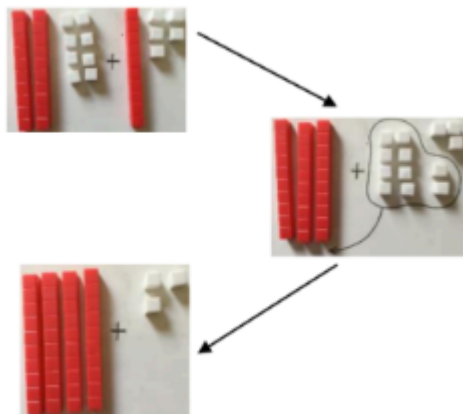
H	T	O
8 9	2 3 4	1 1
10	5 6	2 2 2

H	T	O
100 109	10 10 10 10	1 1
100	10 10	1 1 1

$$\begin{array}{r} 242 \\ + 123 \\ \hline 365 \end{array}$$
$$\begin{aligned} 27 + 5 &= 20 + 7 + 5 \\ &= 20 + 10 + 2 \\ &= 32 \end{aligned}$$
$$\begin{array}{r} 20 + 7 \\ + 5 \\ \hline 20 + 12 \end{array}$$
$$\begin{array}{r} 27 \\ + 5 \\ \hline 32 \end{array}$$

(expanded)

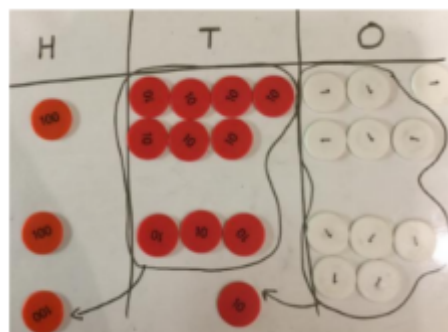
TO + TO using Base 10



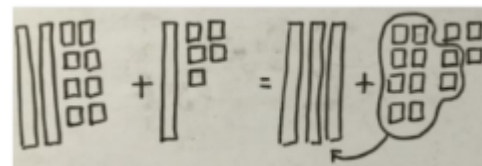
Move onto using place value

counters. HTO + HTO using place

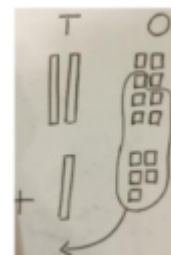
value counters



TO + TO drawing Base 10



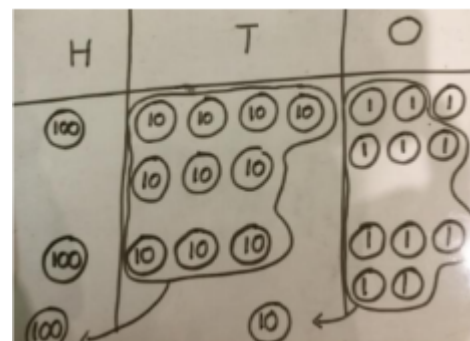
Or



And representing by drawing in a place value frame.

HTO + HTO drawing place value

counters



$$28 + 15 = 20 + 10 + 8 + 5$$

$$= 20 + 10 + 10 + 3$$

$$= 43$$

When ready, moving onto:

$$\begin{array}{r} 20 + 8 \\ + 10 + 5 \\ \hline 30 + 13 \end{array}$$

and/or

$$\begin{array}{r} 28 \\ + 15 \\ \hline 43 \\ \hline \end{array}$$

(expanded)

Add ones, then tens and hundreds

$$\begin{array}{r} 100 + 70 + 6 \\ + 100 + 30 + 5 \\ \hline 200 + 100 + 11 \end{array}$$

(expanded)

$$\begin{array}{r} 176 \\ + 135 \\ \hline 311 \\ \hline \end{array}$$

and/or

Add decimals with up to 3 decimal places

Progress to numbers with digits in different place value columns.
Encourage children to check that they have lined up the columns correctly

I do/do not need to make an exchange because ...

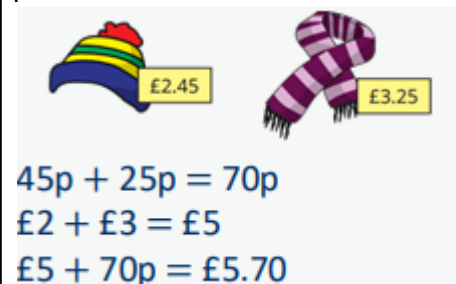


		3	1	0	8
	+	2	1	5	4
		5	2	6	2
					1

		1	5	0	2	7	
	+		9	5	8		
		2	4	6	0	7	
		1	1				

Add decimal numbers in the context of money
Emphasis on partitioning and use of number lines rather than formal written calculations

... pence + ... pence = ... pence ... pounds + ... pounds = ...



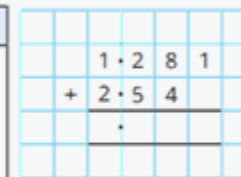
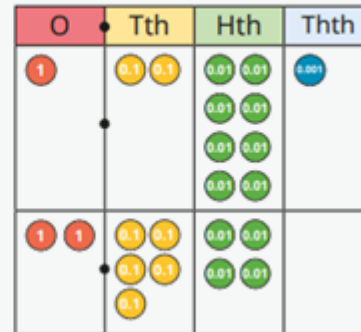
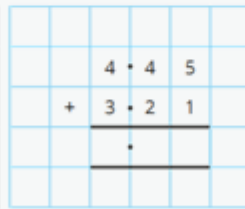
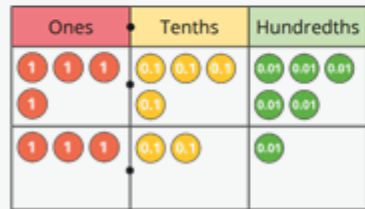
£3.25 can be partitioned into £3 + 20p + 5p or into any possible parts and children need to label each part on the number line.



Add decimals with up to 2 decimal places

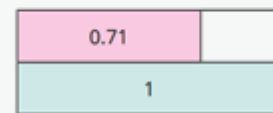
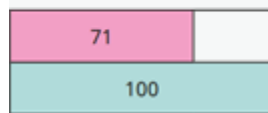
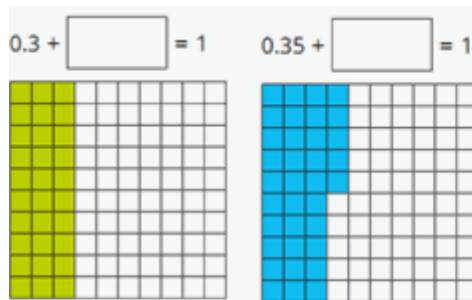
Progress from the same number of decimal places to a different number of decimal places, and from no exchange to exchange.

I do/do not need to make an exchange because ...
I can exchange 10 ... for 1 ...



Complements to 1

Pairs of numbers with up to 3 decimal places which total 1. Encourage children to make links with bonds to 10 and complements to 100 and 1,000.



$$4 + 6 = 10$$

$$44 + 56 = 100$$

$$444 + 556 = 1,000$$

$$0.4 + 0.6 = 1$$

$$0.44 + 0.56 = 1$$

$$0.444 + 0.556 = 1$$

Calculate the duration of events

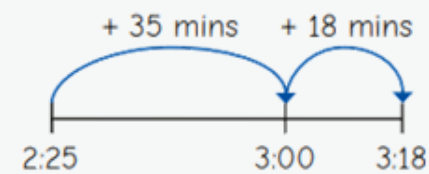
Find durations of time between a given start and end point. Children will need to calculate complements to 60.

From ... to ... o'clock is ... minutes. From ... o'clock to ... is ... minutes. The total time taken is ... minutes.



4:25
start

4:55
finish

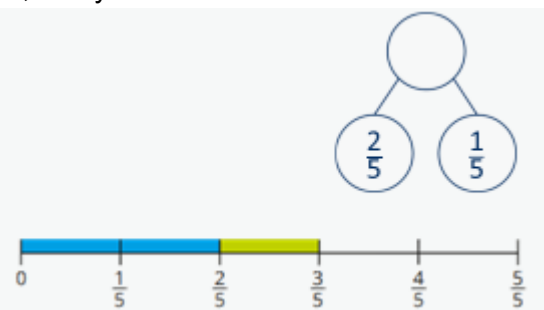
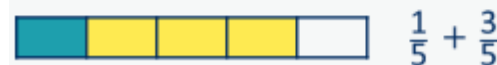


Children can make smaller steps on the number line e.g. 10 min or 5 min

Add fractions with the same denominator within 1 whole

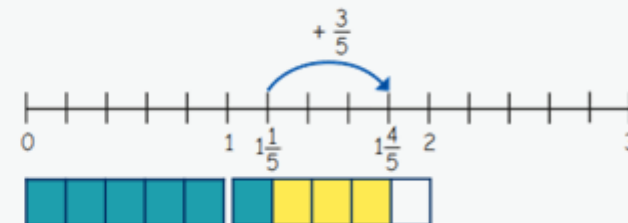
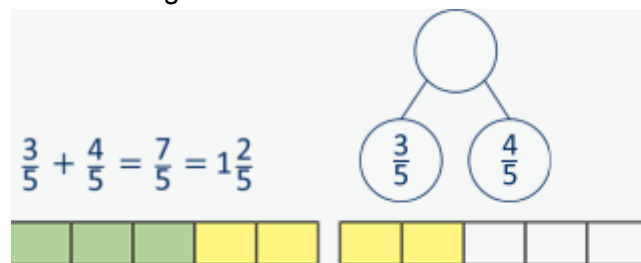
Make links with known facts.

When adding fractions with the same denominator, I only add the numerator. ... fifths + ... fifths = ... fifths



Add fractions and mixed numbers with the same denominator beyond 1 whole

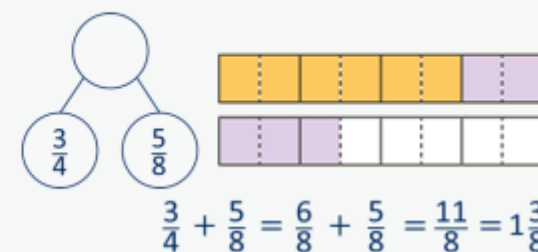
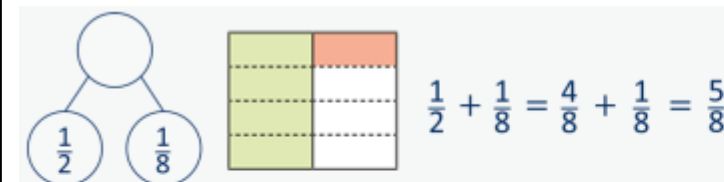
When adding fractions with the same denominator, I only add the numerator. ... fifths + ... fifths = ... fifths



Label the steps on the number line

Add fractions with denominators that are a multiple of one another. Encourage children to convert fractions to the same denominator before adding. Progress from adding fractions within 1 whole to adding fractions beyond 1 whole.

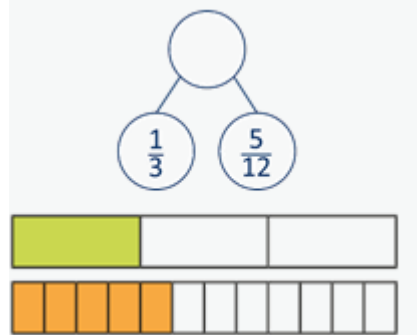
The denominator has been multiplied by ..., so the numerator needs to be multiplied by... for the fractions to be equivalent.



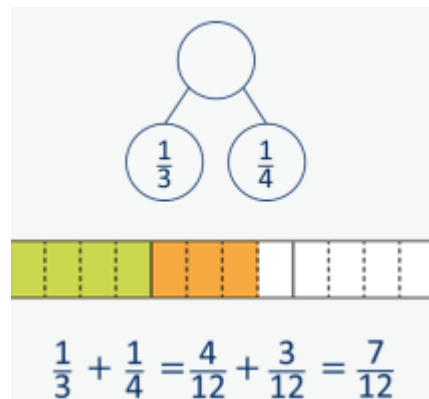
Add fractions

Convert fractions to the same denominator before adding. Progress from fractions where one denominator is a multiple of the other, to any fractions and then to mixed numbers.

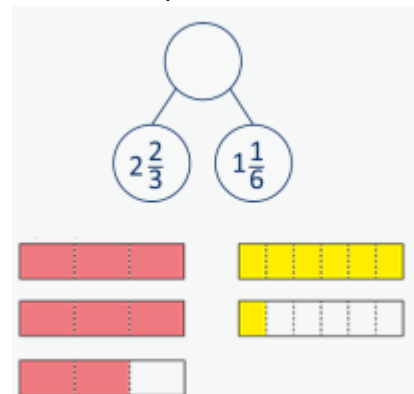
The denominator has been multiplied by ..., so the numerator needs to be multiplied by ...



The lowest common multiple of ... and ... is ...



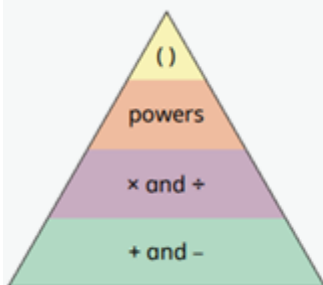
...is made up of ... wholes and ...



Order of operations

Calculations in brackets should be done first. Multiplication and division should be performed before addition and subtraction. *When no brackets are shown and the operations have the same priority, work left to right.

... has greater priority than ..., so the first part of the calculation I need to do is ...



$$(3 + 4) \times 2 = 14$$



$$3 \times 4 + 2 = 14$$



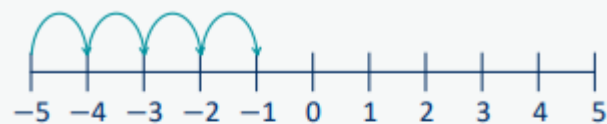
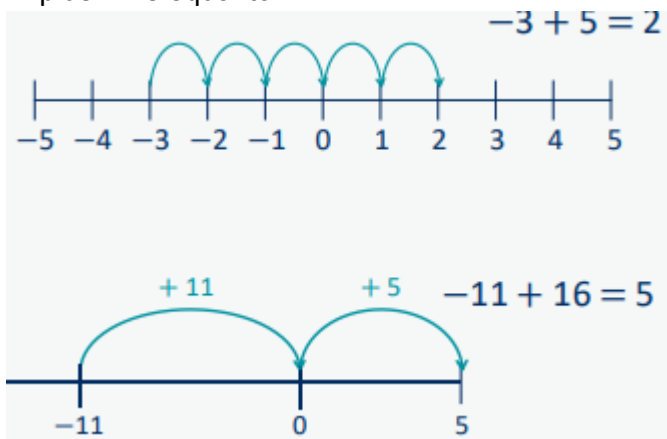
$$3 + 4 \times 2 = 11$$

Negative numbers

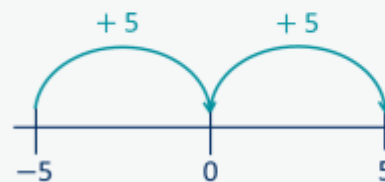
Children add to negative numbers and carry out calculations which cross 0

Label the steps on the number line

... plus ... is equal to ...



The difference between -5 and -1 is 4

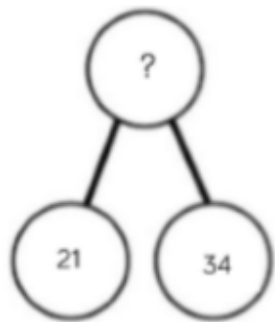


The difference between -5 and 5 is 10

Label the steps on the number line

Conceptual Variation: different ways to ask children to solve $21 + 34$

Suggestions for other models of addition children should use to develop conceptual fluency once they are fluent with methods and procedures.



?	
21	34

Word problems:

In year 3, there are 21 children
and in year 4, there are 34
children.

How many children in total?

$21 + 34 = 55$. Prove it

21

+ 34

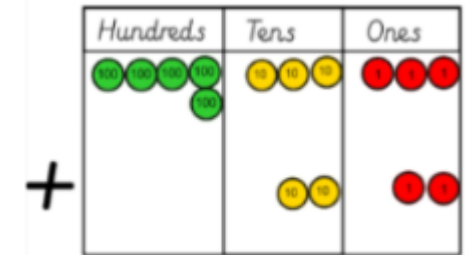
—

$21 + 34 =$

$= 21 + 34$

Calculate the sum of
twenty-one and thirty-four.

Addition represented using
place value counters:



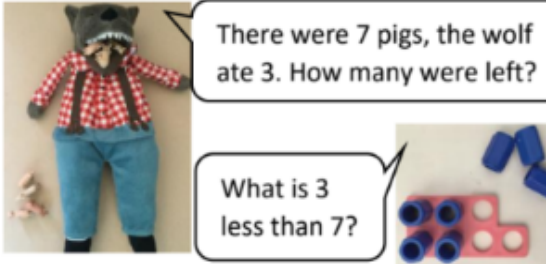
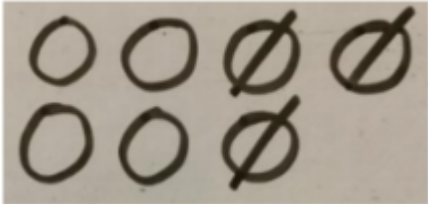
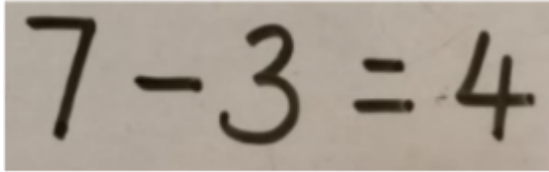
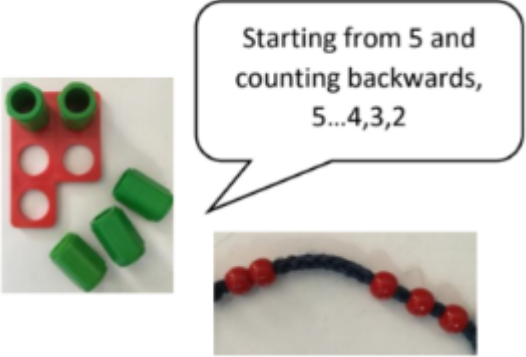
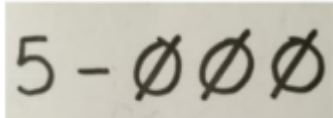
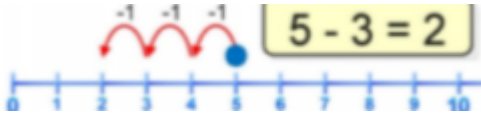
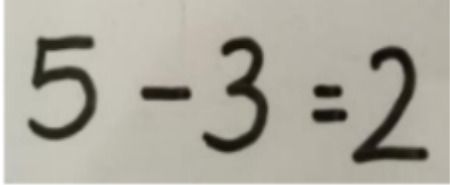
Missing digit problems:

10s	1s
2	1
3	?
?	5

Subtraction

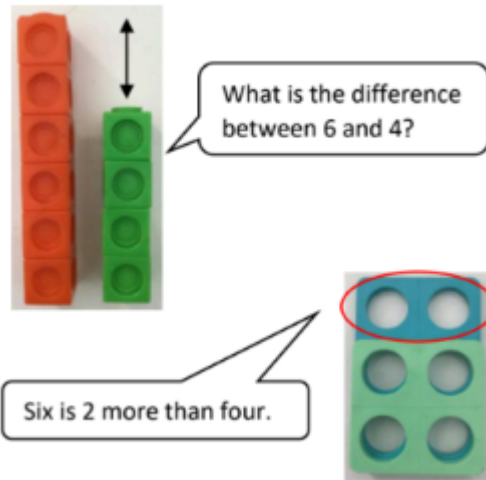
Key language: take away, less than, the difference, subtract, minus, fewer, decrease

	Concrete	Pictorial	Abstract
Finding one less	Taking away or removing an object from the whole – represented using Numicon and pegs, cubes, counters or other familiar objects and resources. Subtraction is introduced through real life situations (e.g. losing something or eating something), through songs (e.g. Five Little Speckled Frogs, Ten Green Bottles) and through stories (e.g. The Three Little Pigs).	Children draw circles to represent the apparatus they are using, crossing one off and counting the remaining circles (or counting backwards, depending on familiarity with numbers & counting).	When ready, children begin to record as an abstract equation (number sentence).

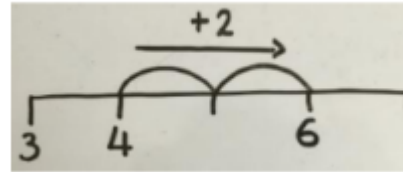
Taking away and removing objects from a whole	Children move onto removing more than one object from the whole, again introduced initially through familiar contexts, stories and songs, but moving onto representing using Numicon and pegs or counters. Children count the number that remain. 	Represented as circles, with the numbers that are being subtracted crossed off. 	Recorded as an equation (number sentence). 
Counting back in ones	Counting backwards as objects are removed from a group, moving onto counting back along a beadbar, numbertrack or numberline. 	The initial amount is recorded as a digit and the amount being taken away is recorded as circles. Each circle is crossed off as children count backwards in ones.  Children are also shown how to count back in ones along a number line . 	Recorded as an equation (number sentence). 

Finding the difference

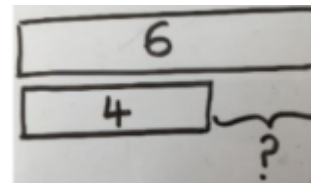
Comparing amounts and objects to find the difference using cubes or Numicon.



Counting on using a number line to find the difference between two numbers. Children are encouraged to begin to draw their own number lines and to realise that they do not need to start from 0 each time.

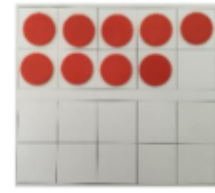
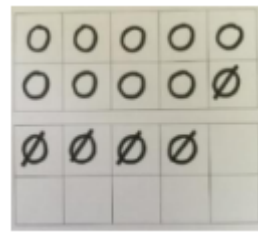
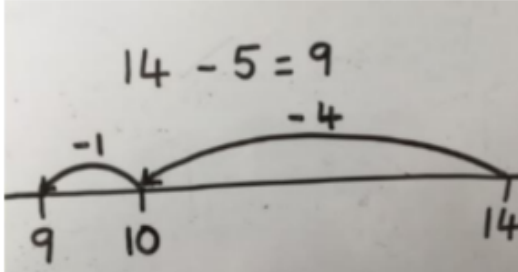
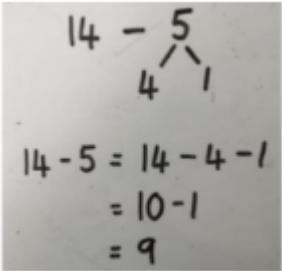
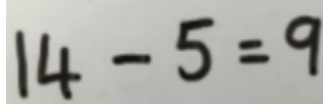
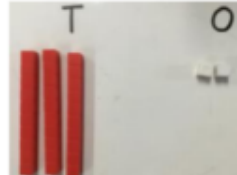
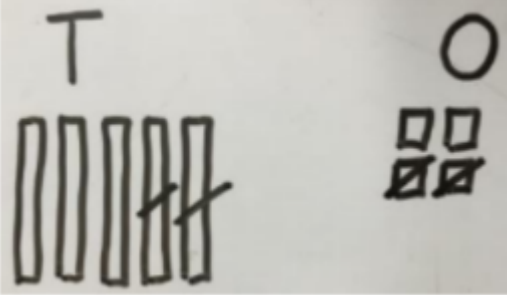
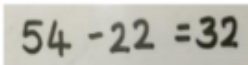
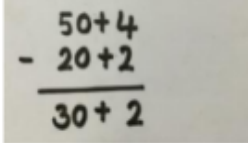
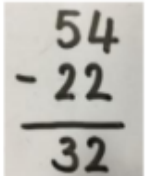


or drawing a comparison bar model.



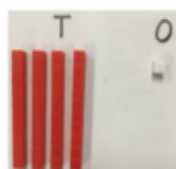
Recorded as an equation (number sentence).

$$6 - 4 = 2$$

Making ten Before this stage, children will make numbers using ten frames and taking away (subtracting) physically.	Using 10 frames to calculate $14 - 5$.   	Children represent the ten frames pictorially and discuss their method.  They can then move onto representing on a number line. 	Initially, children show partitioning in their abstract recording to reinforce their understanding from concrete and pictorial work.  When ready, they move onto recording only the equation. 
Column method (no regrouping/ carrying and exchanging)	Using Base 10 to calculate $54 - 22$  	Children draw representations of Base 10 and cross out the amount being subtracted, first ones and then tens. 	Initially record horizontally before moving onto column subtraction – reinforcing that ones need to be subtracted before tens. Horizontal  Expanded column  Column 

Column method (with regrouping/ carrying and exchanging)

Using Base 10 to calculate $41 - 26$



Represent 41 using Base 10.

Carry one ten to the ones column and exchange for ten ones.



Take away ones and then tens.

Using Place Value counters to calculate $234 - 88$



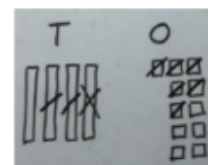
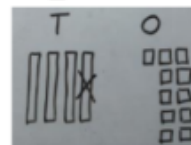
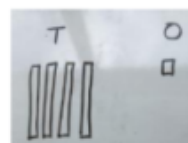
Represent 234 using Place Value counters

Carry a ten to the ones column and exchange. Carry a hundred to the tens and exchange.



Subtract ones and then tens.

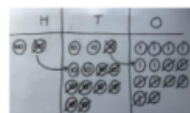
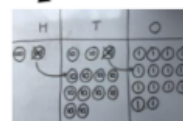
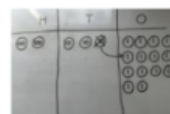
Children draw representations of Base 10, showing carrying and exchanging by crossing out one ten and drawing as ten ones in the ones column. The subtracted ones and tens are then crossed off.



Children draw representations of Place Value counters, showing carrying and exchanging by:

- crossing out one ten and drawing as ten ones in the ones column
- crossing out one hundred and drawing as ten tens in the tens column

The subtracted ones and tens are then crossed off.



Initially record horizontally, partitioning numbers and relating back to the part-whole model. Recombining as the final step to find the answer.

$$\begin{array}{r} 41 - 26 = 15 \\ \begin{array}{cc} 30 & 11 \\ \backslash & / \end{array} \\ \begin{array}{cc} 30 - 20 = 10 \\ 11 - 6 = 5 \end{array} \end{array}$$

Move onto expanded method of column subtraction (if needed), and then formal written/compact method.

$$\begin{array}{r} 30 \\ 40 + 1 \\ - 20 + 6 \\ \hline 10 \quad 5 \end{array}$$

$$\begin{array}{r} 341 \\ - 26 \\ \hline 15 \end{array}$$

Use expanded method initially, moving onto a formal written/compact method.

$$\begin{array}{r} 234 - 88 = 146 \\ \begin{array}{ccc} H & T & O \\ 200 & 30 & 14 \\ - & 80 & + 8 \\ \hline 100 & 40 & 6 \end{array} \end{array}$$

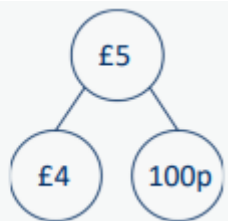
$$\begin{array}{r} 234 \\ - 88 \\ \hline 146 \end{array}$$

Subtract decimal numbers in the context of money

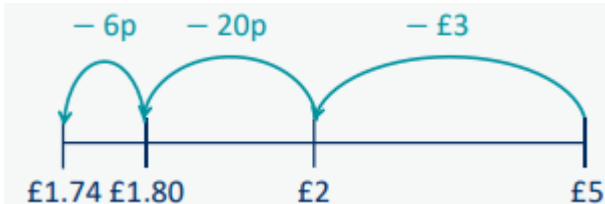
Emphasis here is on partitioning and use of number lines rather than formal written calculations.

I can partition £... into £... and 100p £... - £... = £... 100p - ...p = ...p

$$\begin{aligned} \text{£}5 - \text{£}3.26 \\ \text{£}4 - \text{£}3 &= \text{£}1 \\ 100\text{p} - 26\text{p} &= 74\text{p} \\ \text{£}5 - \text{£}3.26 &= \text{£}1.74 \end{aligned}$$

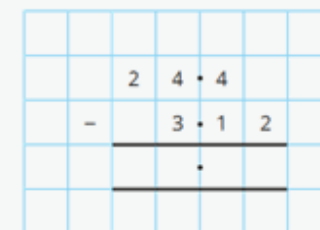
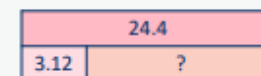
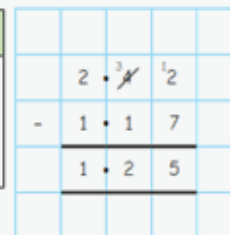
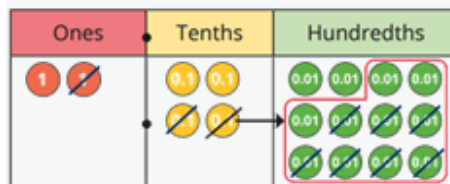


£3.26 can be partitioned into £3 + 20p + 6p



Subtract decimals with up to 2 decimal places

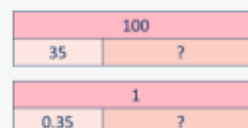
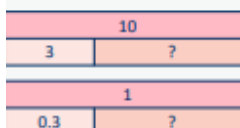
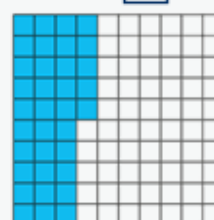
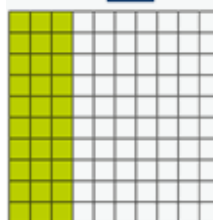
Progress from the same number of decimal places to a different number of decimal places and from no exchange to exchange.



Complements to 1

Encourage children to make links with bonds to 10 and complements to 100 and 1,000 when finding a missing part or subtracting from 1

$$0.3 + \square = 1 \quad 0.35 + \square = 1$$



$$10 - 4 = 6$$

$$1 - 0.4 = 0.6$$

$$100 - 44 = 56$$

$$1 - 0.44 = 0.56$$

$$1,000 - 444 = 556$$

$$1 - 0.444 = 0.556$$

Subtract decimals with up to 3 decimal places

Progress from the same number of decimal and whole number places to a different number of decimal and whole number places.

I do/do not need to make an exchange because ...

		6	~~7~~	13
	-	1	3	4
		5	3	9

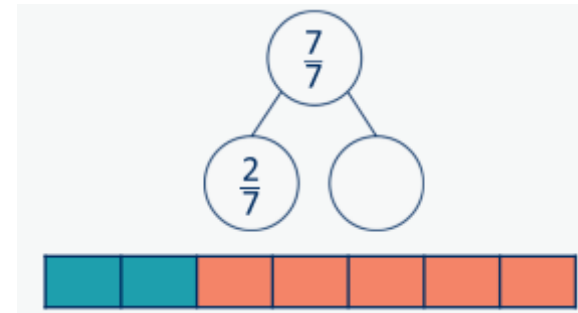
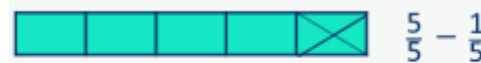
O	Tth	Hth	Thth
0	9	7	5

		~~1~~	~~5~~	15
	-	0	6	4
		0	9	7

Subtract fractions with the same denominator within 1 whole

Make links with known facts.

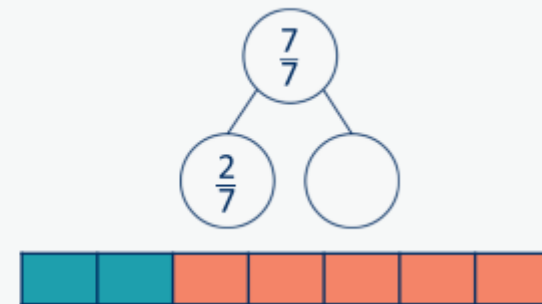
When subtracting fractions with the same denominator, I only subtract the numerator. ... fifths - ... fifths = ... fifths



Subtract fractions with the same denominator within 1

whole Make links with known facts.

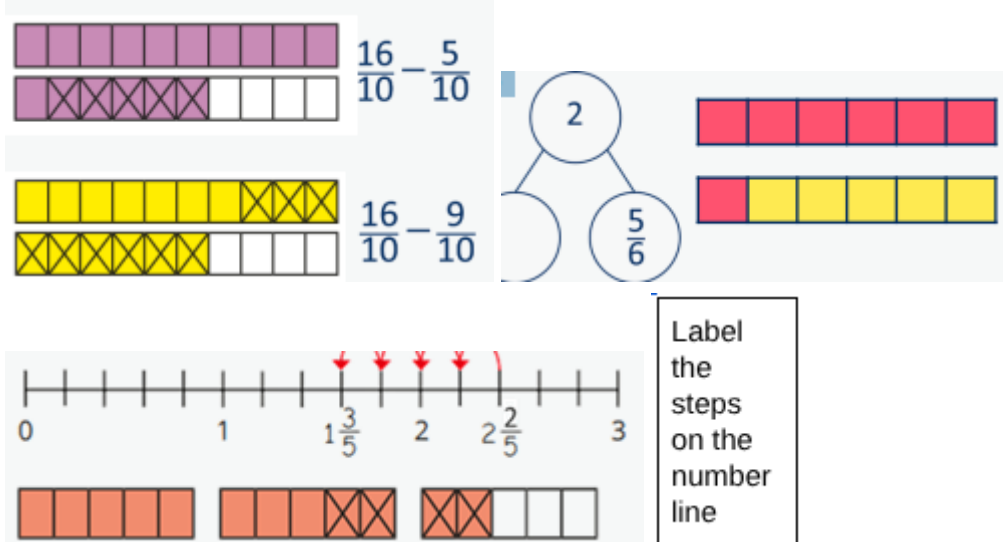
When subtracting fractions with the same denominator, I only subtract the numerator.
... fifths - ... fifths = ... fifths



Subtract fractions and mixed numbers with the same denominator

Include subtracting fractions from wholes.

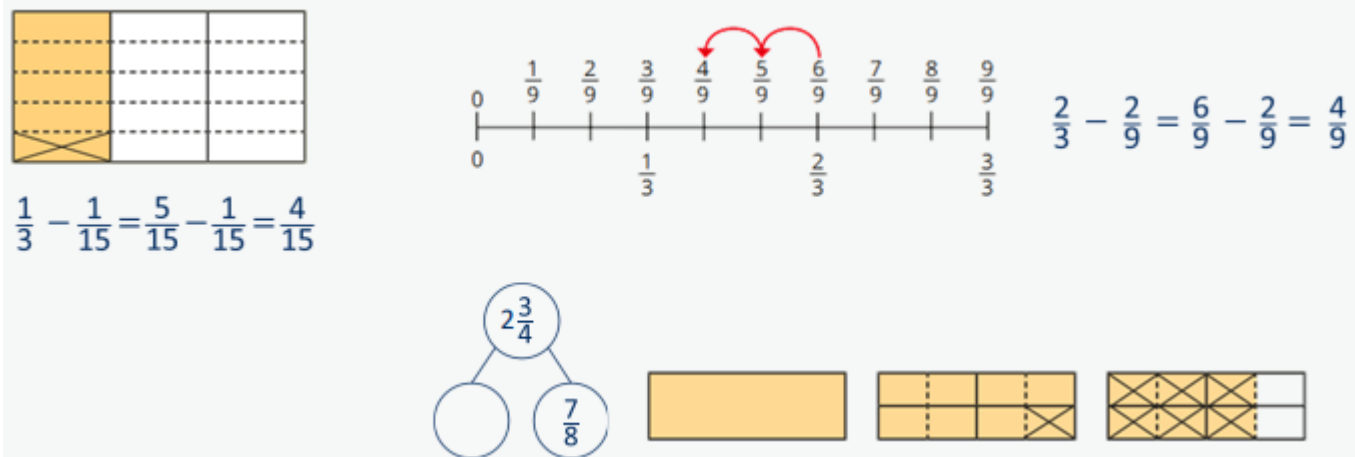
When subtracting fractions with the same denominator, I only subtract the numerator. ... tenths – ... tenths = ... tenths



Subtract fractions with denominators that are a multiple of one another

Convert fractions to the same denominator before subtracting. Progress from subtracting fractions within 1 whole to subtracting from a mixed number.

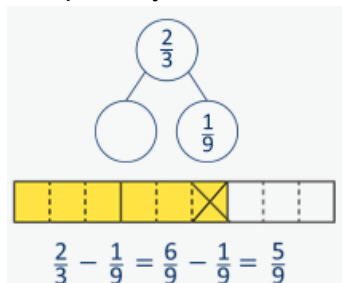
The denominator has been multiplied by ..., so the numerator needs to be multiplied by... for the fractions to be equivalent.



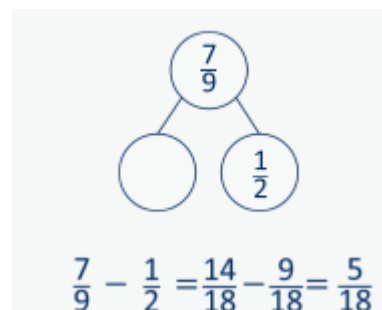
Subtract fractions

Convert fractions to the same denominator before subtracting. Progress from fractions where one denominator is a multiple of the other, to any fractions and then subtracting from a mixed number

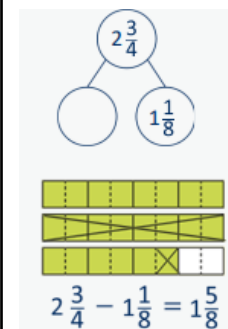
The denominator has been multiplied by ..., so the numerator needs to be multiplied by...



The lowest common multiple of ... and ... is ...



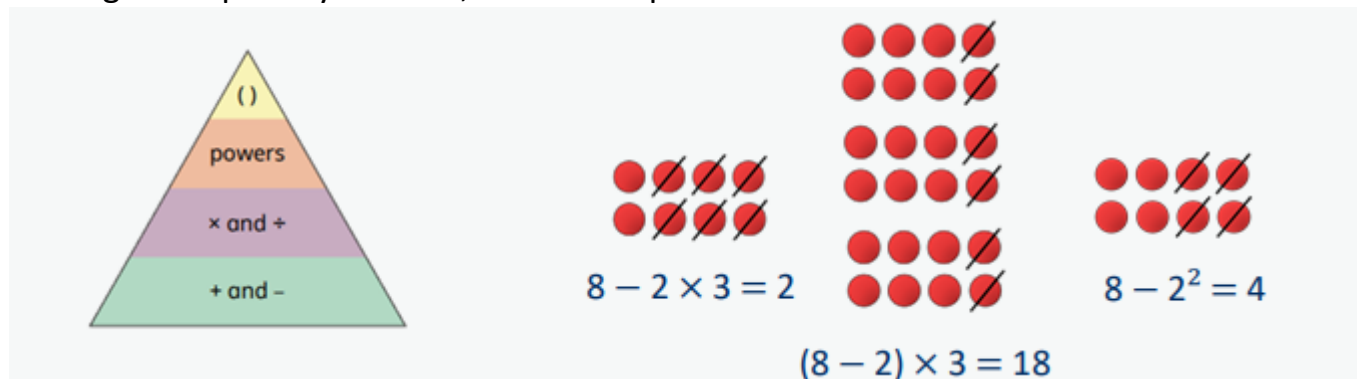
... is made up of ... wholes and ...



Order of operations

Children learn the order of priority for operations in a calculation. Calculations in brackets should be done first. Multiplication and division should be performed before addition and subtraction.

... has greater priority than ..., so the first part of the calculation I need to do is ...



Negative numbers

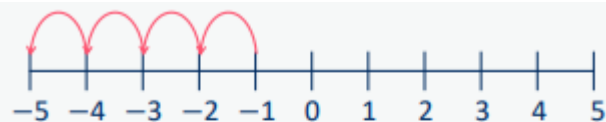
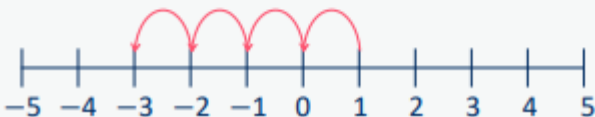
Children subtract from positive and negative numbers and calculate intervals across 0

... minus ... is equal to ...

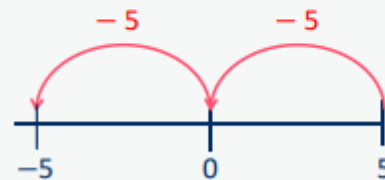
$$-1 - 4 = -5$$



$$1 - 4 = -3$$



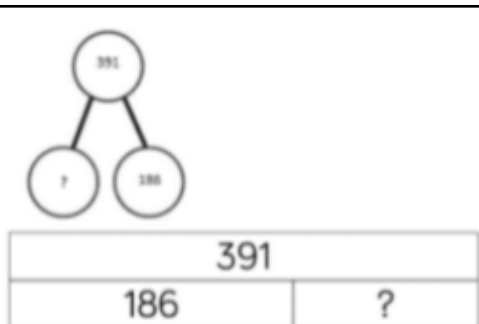
The difference between -5 and -1 is 4



The difference between 5 and -5 is 10

Conceptual Variation: different ways to ask children to solve 391-186

Suggestions for other models of subtraction children should use to develop conceptual fluency once they are fluent with methods and procedures.



Raj spent £391, Timmy spent £186. How much more did Raj spend?

Calculate the difference between 391 and 186.

$$\boxed{} = 391 - 186$$

$$\begin{array}{r} 391 \\ -186 \\ \hline \end{array}$$

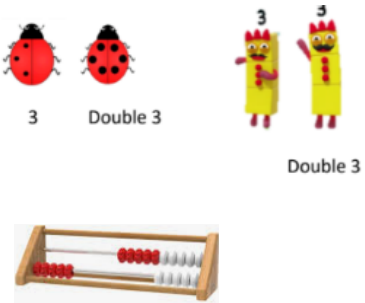
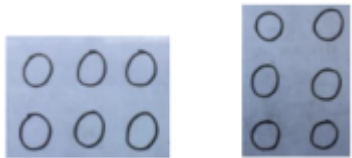
What is 186 less than 391?

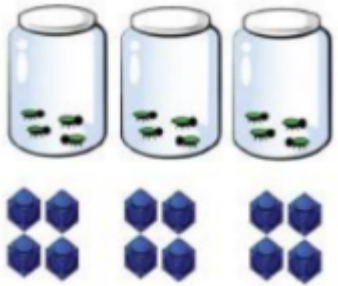
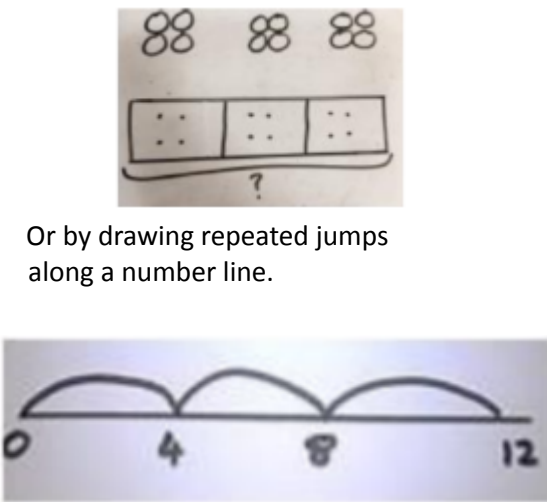
Missing digit calculations

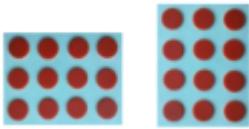
$$\begin{array}{r} 39\boxed{} \\ -\boxed{}\boxed{}6 \\ \hline \boxed{}05 \end{array}$$

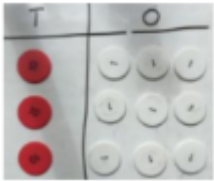
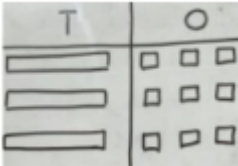
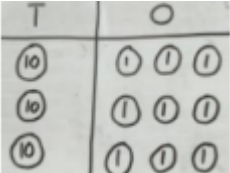
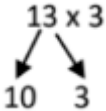
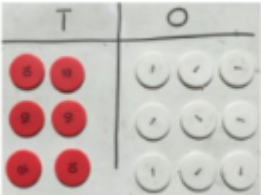
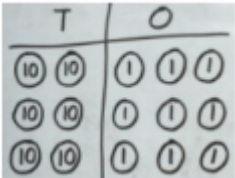
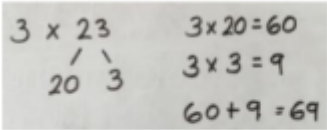
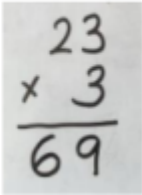
Multiplication

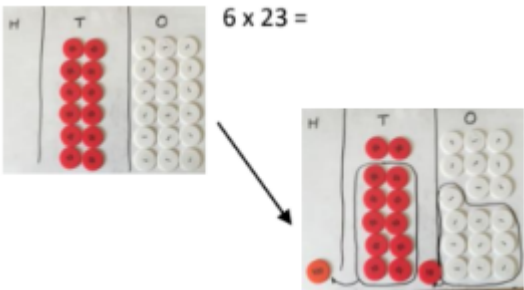
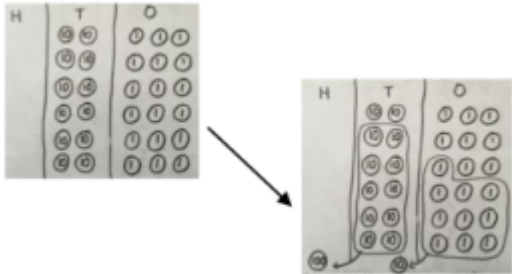
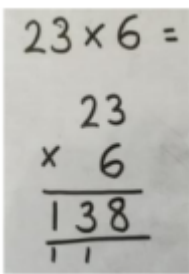
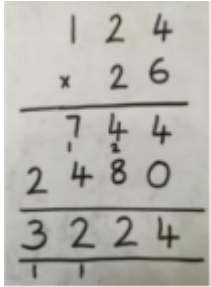
Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups

	Concrete	Pictorial	Abstract
Doubling	In practical activities, children learn to make and show double of a number.  rekenrek	Children record pictorially and are encouraged to set out as an array and to explore if orientation affects the final amount. 	At appropriate stages, children learn to represent doubling as addition or multiplication: $3 + 3 = 6$ $3 \times 2 = 6$

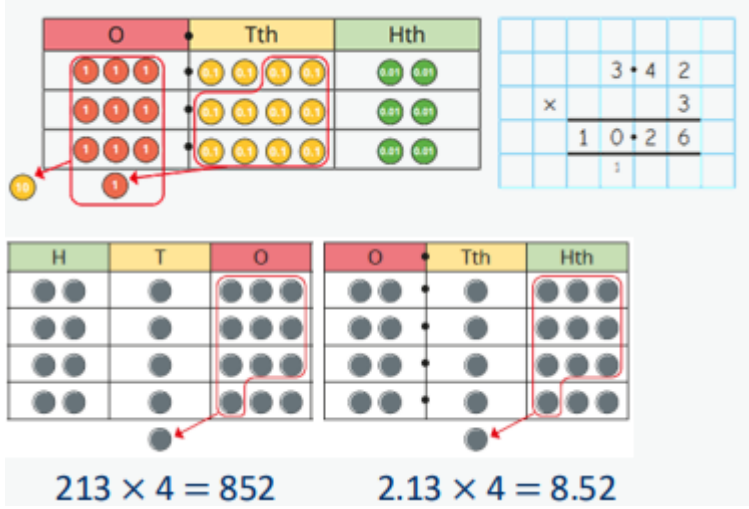
Recognising and making equal groups & Repeated addition	Children recognise and can make equal groups of objects. They use stem sentences to describe the groupings: There are 3 equal groups with 4 in each group. 	Children represent pictorially through drawing resources and bar models.  Or by drawing repeated jumps along a number line.	The total amount is represented by repeated addition and the use of a multiplication sign is introduced. $4 + 4 + 4 = 12$ $3 \times 4 = 12$
--	---	---	--

Using arrays to demonstrate Commutativity Before this stage, children need to understand repeated addition	Children use arrays to illustrate and explore commutativity.  $3 \times 4 = 4 \times 3$	They represent arrays pictorially. 	Arrays can be used by children to derive and write a range of calculations e.g. $3 + 3 + 3 + 3 = 12$ $12 = 4 + 4 + 4$ $12 = 4 \times 3$ $3 \times 4 = 12$
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Partitioning to multiply	Initially, children use Base 10 to explore multiplying a two digit number by a single digit number.  $3 \times 13 =$ When secure with this, they move onto using place value counters.  $3 \times 13 =$	They represent their work pictorially drawing Base 10 representations to begin with...  ... moving onto representing place value counters 	Children use partitioning and known number facts and are encouraged to record the steps they have taken to calculate the answer.  $10 \times 3 = 30$ $3 \times 3 = 9$ $30 + 9 = 39$
Column multiplication	Children use place value counters to represent the calculation.  $4 \times 23 =$	Place value counters are represented pictorially . 	Children record, using an expanded method, to demonstrate their understanding.  

Formal column method	Children use place value counters to represent the calculation.	Place value counters are represented pictorially.	Formal written method.
			 

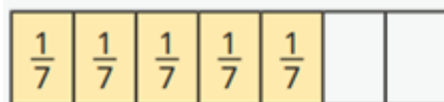
Once children start to multiply 4 digit numbers by 2 digit numbers etc., they should be confident with the abstract.

Multiply decimals by integers This is the first time children multiply decimals by numbers other than 10, 100 or 1,000 Encourage them to make links with known facts and whole number multiplication.	I know that ... $\times$... = ..., so I also know that ... $\times$... = ... 	I need to exchange 10 ... for 1 ... 
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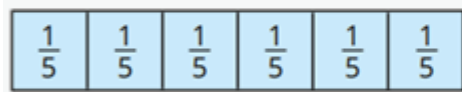
Multiply fractions by a whole number Make links with	To multiply a fraction by an integer, I multiply the numerator by the integer and the denominator remains the same.
--	---

repeated addition.

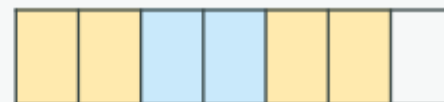
E.g. $\frac{1}{5} \times 4 = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$



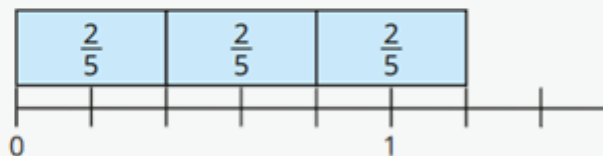
$$\frac{1}{7} \times 5 = \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} = \frac{5}{7}$$



$$\frac{1}{5} \times 6 = \frac{6}{5} = 1\frac{1}{5}$$



$$\frac{2}{7} \times 3 = \frac{2}{7} + \frac{2}{7} + \frac{2}{7} = \frac{6}{7}$$



$$\frac{2}{5} \times 3 = \frac{6}{5} = 1\frac{1}{5}$$

Multiply mixed numbers by a whole number

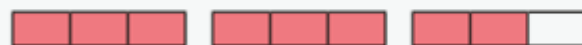
I can partition $\frac{\square}{\square}$ into $\frac{\square}{\square}$ and $\frac{\square}{\square}$



$$2\frac{2}{3} \times 3$$



$$2 \times 3 = 6 \quad \frac{2}{3} \times 3 = \frac{6}{3} = 2$$



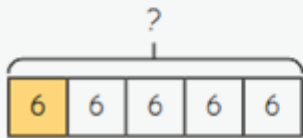
$$2\frac{2}{3} \times 3 = 6 + 2 = 8$$

Find the whole

Children multiply to find the whole from a given part.

If $\frac{1}{\square}$ is ... , then the whole is ... $\times$...

$$\frac{1}{5} \text{ of } \underline{\quad} = 6$$

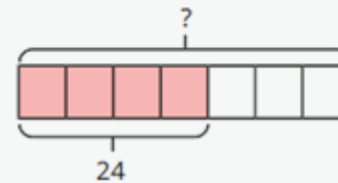


$$5 \times 6 = 30$$

$$\frac{1}{5} \text{ of } \mathbf{30} = 6$$

If $\frac{\square}{\square}$ is ... , then $\frac{1}{\square}$ is ... and the whole is ... $\times$...

$$\frac{4}{7} \text{ of } \underline{\quad} = 24$$



$$\frac{1}{7} = 24 \div 4 = 6$$

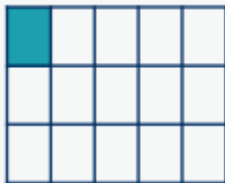
$$7 \times 6 = 42$$

$$\frac{4}{7} \text{ of } \mathbf{42} = 24$$

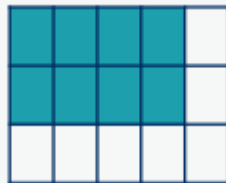
Multiply fractions by fractions

Encourage children to give answers in their simplest form.

When multiplying a pair of fractions, I need to multiply the numerator and multiply the denominator.



$$\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$$



$$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$$



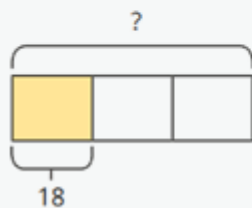
$$\frac{2}{3} \times \frac{3}{5} = \frac{6}{15} = \frac{2}{5}$$

Find the whole

Children multiply to find the whole from a given part.

If $\frac{1}{\square}$ is ... , then the whole is ... $\times$...

$$\frac{1}{3} \text{ of } \underline{\quad} = 18$$

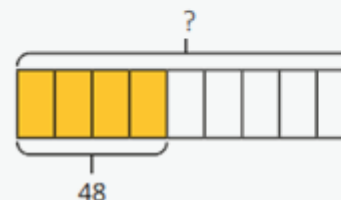


$$18 \times 3 = 54$$

$$\frac{1}{3} \text{ of } \mathbf{54} = 18$$

If $\frac{\square}{\square}$ is ... , then $\frac{1}{\square}$ is ... and the whole is ... $\times$...

$$\frac{4}{9} \text{ of } \underline{\quad} = 48$$



$$\frac{1}{9} = 48 \div 4 = 12$$

$$9 \times 12 = 108$$

$$\frac{4}{9} \text{ of } \mathbf{108} = 48$$

Scaling

Children focus on multiplication as scaling (... times the size) as opposed to repeated addition.

There are times as many ... as ...



There are 3 times as many triangles as circles.

... is ... times the size of is ... times the length/height of ...



Miss Smith is twice the height of Jo.

Calculate percentages

Children first learn how to find 1%, 10%, 20%, 25% and 50% before using multiples of these amounts to find any percentage.

There are ... lots of ... % in 100% To find ... %, I need to divide by ...

100%			
50%		50%	
25%	25%	25%	25%

$$50\% \text{ of } \dots = \dots \div 2$$

$$25\% \text{ of } \dots = \dots \div 4$$

... % is made up of ... %, and ... %

100%									
10%	10%	10%	10%	10%	10%	10%	10%	10%	10%

To find 30%, I can find 10% and then multiply it by 3

To find 23%,

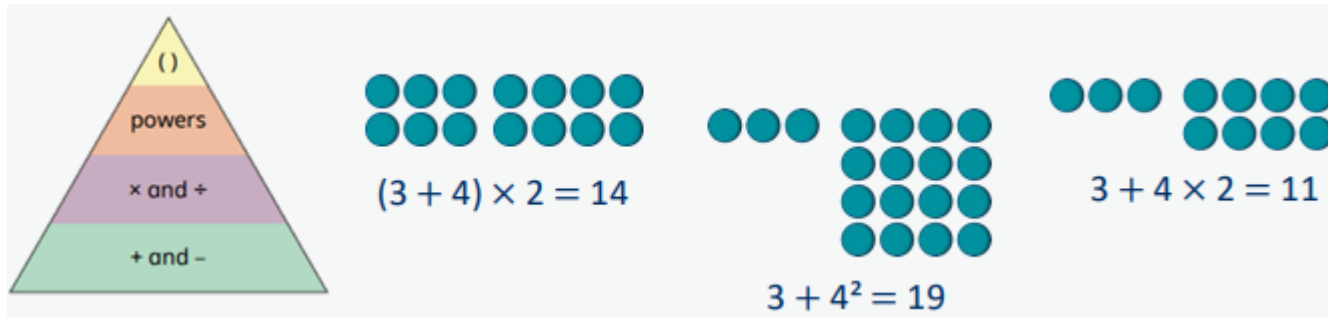
I can use $10\% \times 2$ and $1\% \times 3$ To find 99%,

I can find 1%, then subtract from 100%

Order of operations

Calculations in brackets should be done first. Multiplication and division should be performed before addition and subtraction.

... has greater priority than ..., so the first part of the calculation I need to do is ...



Conceptual Variation: different ways to ask children to solve 6×23

Suggestions for other models of multiplication children should use to develop conceptual fluency once they are fluent with methods and procedures.

23	23	23	23	23	23

Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?

With the counters, prove that $6 \times 23 = 138$

Find the product of 6 and 23.

$$6 \times 23 =$$

$$\boxed{} = 6 \times 23$$

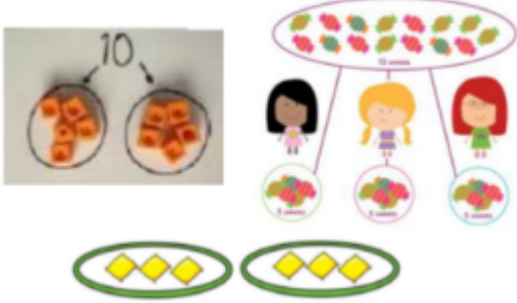
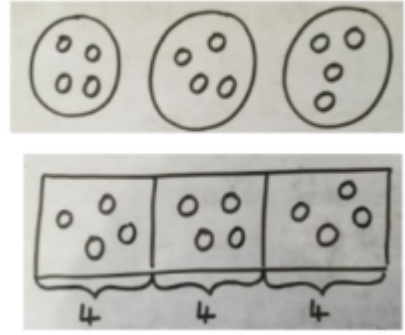
$$\begin{array}{r} 6 \\ \times 23 \\ \hline \end{array} \quad \begin{array}{r} 23 \\ \times 6 \\ \hline \end{array}$$

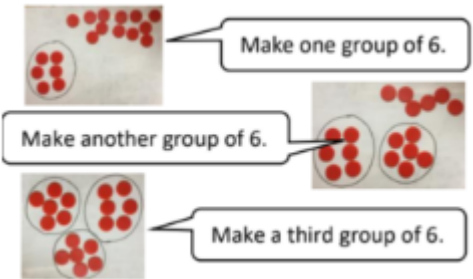
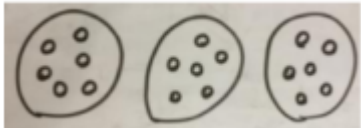
What is the calculation?
What is the product?

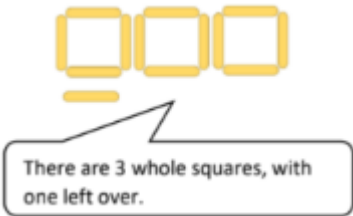
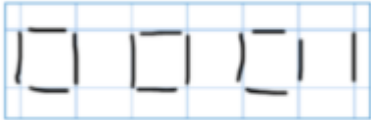
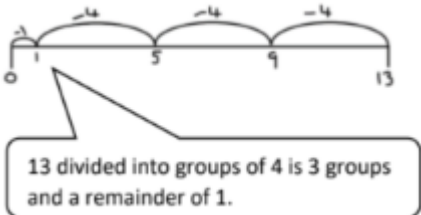
100s	10s	1s

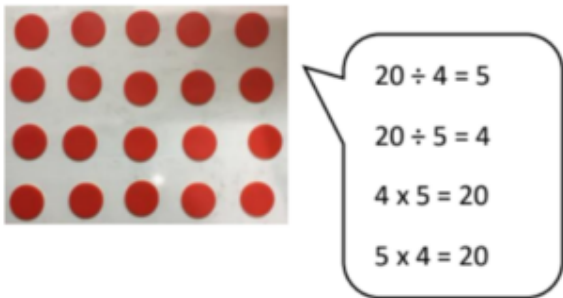
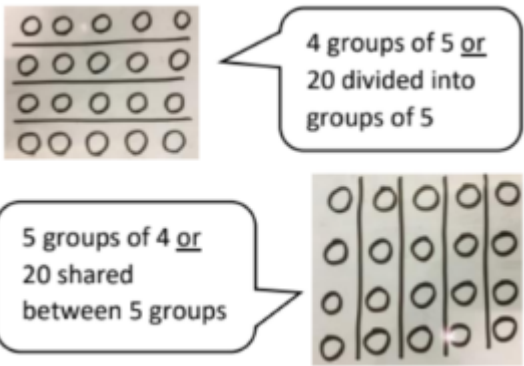
Division

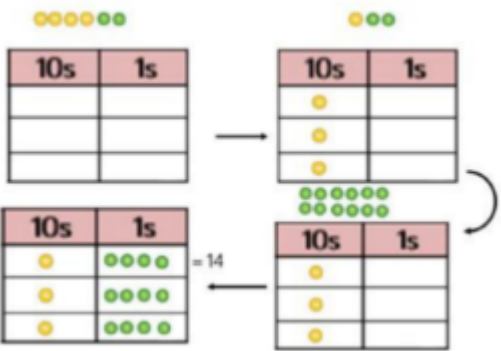
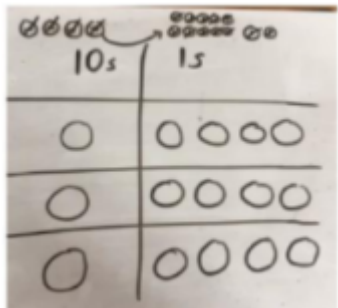
Key language: share, group, divide, divided by, half

	Concrete	Pictorial	Abstract
Sharing	Children are introduced to sharing through practical applications such as sharing biscuits, sweets or fruit. They investigate sharing between different amounts of groups and explore how numbers such as 12 can be shared in several different ways. 	Children draw pictorial representations, either in groups or as a bar model.  Stem sentence: ____ divided between ____ is ____ each.	Children move onto recording the abstract division equation when they are ready. Regular use of stem sentences has helped to prepare them for this. $6 \div 2 = 3$ $10 \div 2 = 5$ $15 \div 3 = 5$

Grouping	Grouping is introduced through practical applications e.g. there are 18 children who need to get into teams of 6. How many teams will there be? Children can work this out practically using counters or cubes. 	Children represent grouping through drawing the individual groups (and the remainder if there is one).  Stem sentence: ___ divided into groups of ___ is ___ groups.	Children record the abstract equation when they are ready. Use of the stem sentence has helped to prepare them for this. $18 \div 6 = 3$
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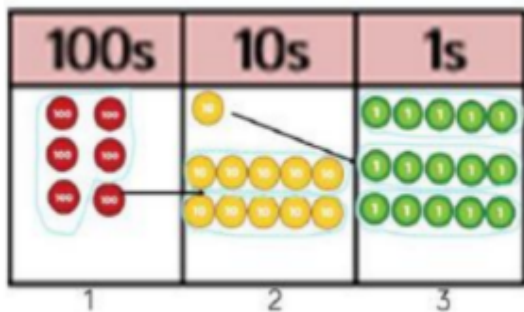
Repeated subtraction	Children are shown how grouping can also be seen as repeated subtraction (the inverse operation of repeated addition in multiplication). e.g. $13 \div 4$ Lollipop sticks, matches or straws could be used as apparatus, with the groups making squares because we are dividing by 4. 	The lollipop sticks can be represented pictorially,  and on a number line. 	The abstract equation is recorded. $13 \div 4 = 3 \text{ remainder } 1$
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Division with arrays	Division can be linked to multiplication by creating an array and thinking about the equations that can be created. 	Children draw arrays and use vertical and horizontal lines to split the array into different groups to make multiplication and division equations. 	Multiplication and division fact families can be derived and recorded. $4 \times 5 = 20$ $20 = 4 \times 5$ $5 \times 4 = 20$ $20 = 5 \times 4$ $20 \div 4 = 5$ $5 = 20 \div 4$ $20 \div 5 = 4$ $4 = 20 \div 5$
-----------------------------	--	--	---

Sharing using place value Counters Link understanding of division to times tables	Using place value counters to share. $42 \div 3$ 	Children represent the place value counters pictorially. 	Children are able to write calculations to show the process: $42 \div 3$ $42 = 30 + 12$ $30 \div 3 = 10$ $12 \div 3 = 4$ $10 + 4 = 14$
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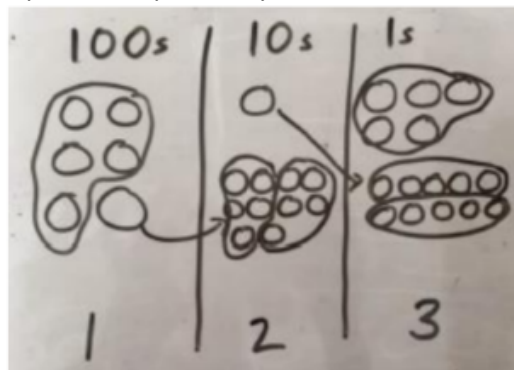
Short division

Using place value counters to group.
 $615 \div 5$



1. Make 615 with the place value counters
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

The place value counters can be represented pictorially.



The calculation is recorded as a short Division.

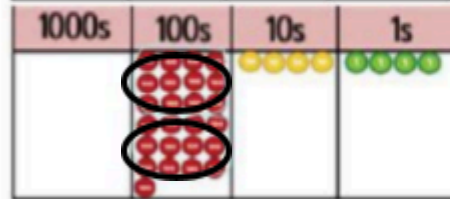
$$\begin{array}{r} 123 \\ 5 \overline{) 615} \end{array}$$

Long Division

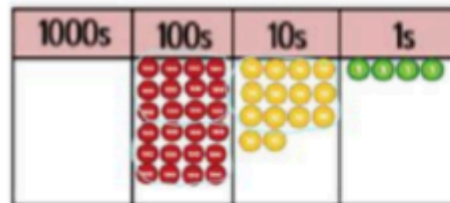
Long division using place value counters – children should be confident with abstract short division before beginning long division. $2544 \div 12$



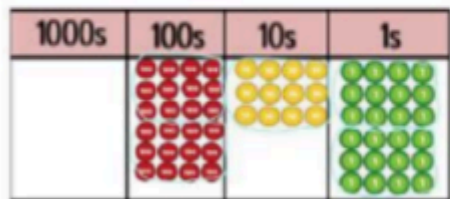
We can't group 2 thousands into groups of 12 so will exchange them.



We can group 24 hundreds into groups of 12 which leaves us with 1 hundred



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.



After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 groups of 12, which leaves no remainder.

$$\begin{array}{r} 02 \\ 12 \overline{) 2544} \\ \underline{24} \\ 1 \end{array}$$

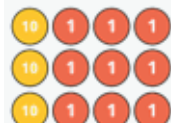
$$\begin{array}{r} 021 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$

$$\begin{array}{r} 0212 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

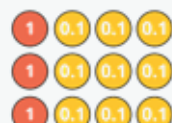
Divide decimals by integers

This is the first time children divide decimals by numbers other than 10, 100 or 1,000

I know that $\dots \div \dots = \dots$, so I also know that $\dots \div \dots = \dots$



$$39 \div 3 = 13$$

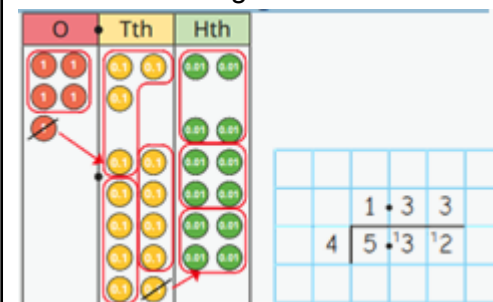


$$3.9 \div 3 = 1.3$$



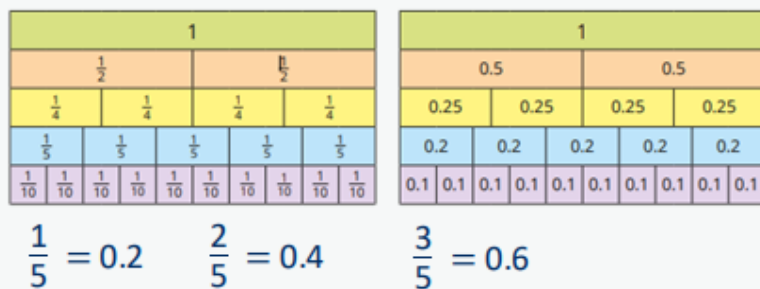
$$0.39 \div 3 = 0.13$$

I need to exchange 1 ... for 10 ...



Decimal and fraction equivalents

The fraction ... is equivalent to the decimal ...



$\frac{3}{4}$ is equal to $\frac{75}{100}$

$\times 25$

$\frac{3}{4} = \frac{75}{100} = 0.75$

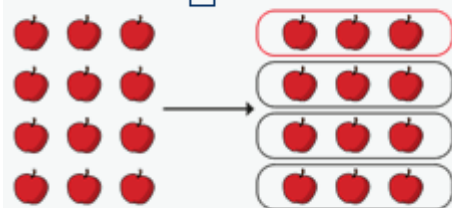
$\times 25$

Unit fractions of a set of objects

Bar models are useful to show the link between division and fractions, for example, dividing by 3 and finding a third.

The whole is divided into ... equal parts.

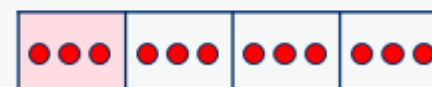
Each part is $\frac{1}{\square}$ of the whole.



$\frac{1}{4}$ of 12 apples is 3 apples.

One ... of ... is ...

$\frac{1}{4}$ of 12 is 3



$\frac{1}{3}$ of 36 is 12



Unit fractions In Y2 the focus is on finding $\frac{1}{2}$, $\frac{1}{4}$

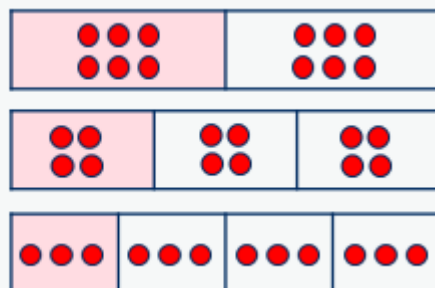
, $\frac{1}{4}$

And $\frac{1}{3}$.

Bar models are useful to show the link between division and finding a fraction.

The objects have been shared fairly into ... groups.

$\frac{1}{\square}$ of ... is ...



There are ... equal parts. There is ... part circled.

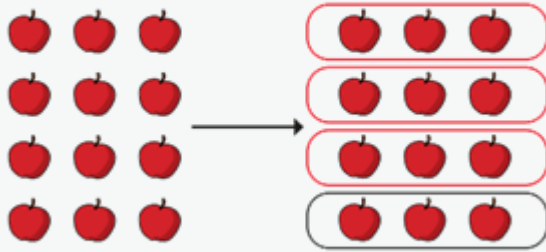
$\frac{1}{\square}$ is circled.



Non-unit fractions of a set of objects Bar models are a useful representation and show the links with division and multiplication.

The whole is divided into ... equal parts.

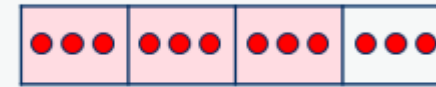
Each part is $\frac{1}{\square}$ of the whole.



$\frac{3}{4}$ of 12 apples is 9 apples.

$\frac{1}{\square}$ of ... is ..., so $\frac{\square}{\square}$ of ... is ...

$\frac{3}{4}$ of 12 is 9



$\frac{2}{3}$ of 36 is 24



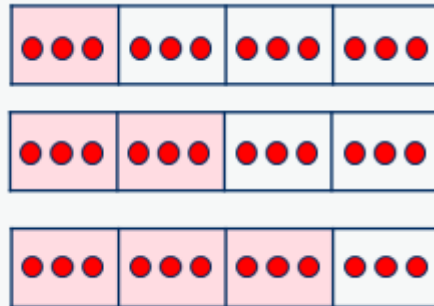
Non-unit fractions

In Y2 the focus is on finding $\frac{2}{4}$ and $\frac{3}{4}$

Prompt children to notice that $\frac{2}{4}$ is equivalent to $\frac{1}{2}$

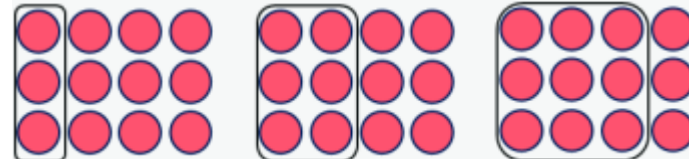
The objects have been shared fairly into ... groups

$\frac{\square}{\square}$ of ... is ...



There are ... equal parts. There are ... parts circled.

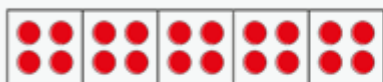
$\frac{\square}{\square}$ is circled.



Fraction of an amount

Bar models support children to understand that to find a fraction of an amount, we divide by the denominator and multiply by the numerator

To find $\frac{\square}{\square}$ of ... , I need to divide by ... and multiply by ...



$$\frac{1}{5} \text{ of } 20 =$$

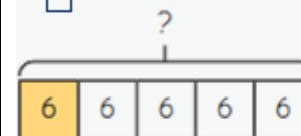
$$\frac{3}{5} \text{ of } 20 =$$



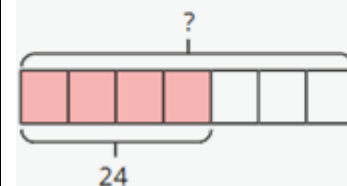
$$\frac{1}{4} \text{ of } 84 =$$

$$\frac{3}{4} \text{ of } 84 =$$

If $\frac{1}{\square}$ is ... , then the whole is ... $\times$...



$$\frac{1}{5} \text{ of } _ = 6$$



$$\frac{4}{7} \text{ of } _ = 24$$

Fraction of an amount

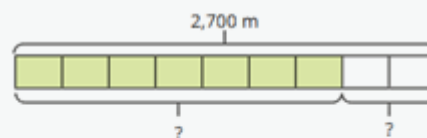
Children divide and multiply to find fractions of an amount. Bar models can still be used to support understanding where needed.

To find $\frac{1}{\square}$ I divide by ...

$$\frac{1}{2} \text{ of } 36 = 36 \div 2$$

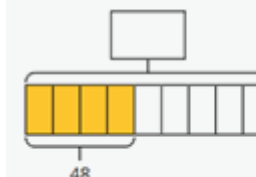
$$\frac{1}{12} \text{ of } 36 = 36 \div 12$$

If $\frac{1}{\square}$ is equal to ..., then $\frac{\square}{\square}$ are equal to ...



$$\frac{7}{9} \text{ of } 2,700 = \frac{1}{9} \text{ of } 2,700 \times 7$$

If $\frac{\square}{\square}$ is equal to ..., then the whole is equal to ...



$$\frac{4}{9} \text{ of } _ = 48$$

Divide a fraction by an integer

This is the first time children divide fractions by an integer.

...ones divided by 2 is ... ones so ... sevenths divided by 2 is ... sevenths.

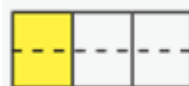


$$\frac{4}{7} \div 4 = \frac{1}{7}$$



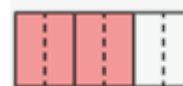
$$\frac{4}{7} \div 2 = \frac{2}{7}$$

I am dividing by ... , so I can split each part into ... equal parts.



$$\frac{1}{3} \div 2 = \frac{1}{6}$$

... is equivalent to ... so ... $\div$... = ... $\div$...



$$\frac{2}{3} = \frac{4}{6}$$

$$\text{so } \frac{2}{3} \div 4 = \frac{4}{6} \div 4 = \frac{1}{6}$$

Calculations involving ratio

Encourage children to see the multiplicative relationship between ratios. They will need to multiply or divide each value by the same number to keep the ratio equivalent. Double number lines and ratio tables help children to see both horizontal and vertical multiplicative relationships.

For every ... , there are ...

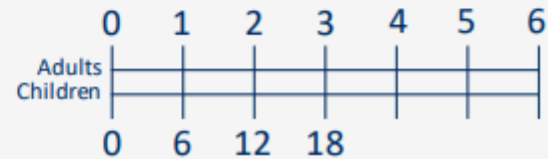
For every 6 children on a school trip, there is 1 adult.

adults 

children 

÷ 6	
Adults	Children
1	6
2	12
3	18
÷ 3	

The ratio of children to adults is 6 : 1



Conceptual Variation: different ways to ask children to solve $415 \div 5$

Suggestions for other models of division children should use to develop conceptual fluency once they are fluent with methods and procedures.

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$$5 \overline{)615}$$

$$615 \div 5 =$$

$$\boxed{} = 615 \div 5$$

What is the calculation?
What is the answer?

